Stochastic Programming for Optimizing Business Supply Chains

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Outline

- Introduction to business supply chains
- Optimizing a configure-to-order (CTO) supply chain
- Overview of stochastic programming (SP)
- Application of SP to CTO problem
- Conclusions
What is a Supply Chain?

- A network of retailers, distributors, transporters, storage facilities and suppliers that participate in the sale, delivery and production of a particular product or service to customers.

- A supply chain may include multiple companies which may be located in multiple countries.
What is Supply Chain Management?

- Supply Chain Management (SCM) refers to the coordination of the ‘flow’ of information, cash, and materials between the ‘nodes’ in a supply chain network.

- **Information Flows**
  - Customer Orders, Demand Forecasts, Inventory Levels, Staffing levels

- **Cash Flows**
  - Inflow: Customer Payments, Sale of Assets
  - Outflow: Purchase of supplies, assets, services

- **Materials**
  - Raw materials, work-in-progress (WIP), finished goods
SCOR Reference Model

Suppliers

SOURCE

MAKE

DELIVER

Customers

PLAN

RETURN
Levels of Supply Chain Planning

- **Strategic**
  - Long term, on the order of years
  - E.g., facility location, new product development, supplier selection

- **Tactical**
  - Medium term, on the order of weeks/months
  - E.g., Demand forecasting, workforce planning, inventory planning

- **Operational**
  - Short term, on the order of days or minutes.
  - E.g., machine scheduling, staff scheduling, order fulfillment
Sales & Operations Meeting

Sales targets → Commitment to Sales

Manufacturing

MRP explosion → Supply Commitment

Suppliers

- sales
- marketing
- accounting
- finance
- manufacturing

DEMAND/SUPPLY PLANNING

Want High Revenue
Want High Profit

NEED SUPPLY = DEMAND
Outline

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A Configurable Product

- A configurable product is **modular** in structure, and is assembled from standardized components.
- e.g., Computers (PCs and Mainframes), cars, homes, travel packages, course schedules.
Configure-to-Order Supply Chains

- In a configure-to-order supply chain:
  - The *quantity* of each component required to assemble a finished product is uncertain prior to receiving the product order. (i.e., *Order Configuration Uncertainty*)
  - When dealing with several configurable products, components may be used by more than one product (i.e., *Component Commonality*)
  - Configure-to-Order systems are more common today, as the practice of mass customization proliferates.
Order Configuration Uncertainty

- Consider a configurable product that may use component A and/or component B.

- Along with component commonality, configuration uncertainty makes supply/demand planning in a CTO system very challenging.
Demand/Supply Planning

- Tactical level planning (32 wk rolling horizon)
Demand/Supply Optimization

- **Explosion Problem**
  - Initial Sales Targets
  - EXPLOSION
  - Optimal Supply Request
- **Implosion Problem**
  - Suppliers
  - IMPLOSION
  - Supply Commitment
  - Optimal Commitment to Sales
Outline

- Introduction to business supply chains
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- Conclusions
Quick Intro to Linear Programming

- Parameters ($c$)
- Decision Variables ($x$)
- Constraints ($Ax = b$)
- Feasible Region
- Objective Function ($\text{max. } c^T x$)
- Optimal solution ($x^*$)
Linear Programming Constraint Matrix
Linear Programming Constraint Matrix
Stochastic Programming

- In Stochastic Programming, one or more set(s) of parameters (e.g., c, A, b) may consist of random variables (discrete or continuous).

- Therefore, the feasibility of any solution, x, may depend on the realization of these random variables.

- A particular combination of realizations of parameters is referred to as a scenario.

- Typically the objective is to maximize or minimize an expected value function.
Two-Stage Stochastic Programs with Recourse

- 2 sets of decision variables
  - 1st stage variables, x
  - 2nd stage variables, y

- In the first stage, random parameter realizations are unknown, and x is decided.

- In the second stage, random parameters are realized and y is decided, given x.

- Multi-stage SPs are extensions of the 2 stage SP with non-anticipatory constraints.
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Demand/Supply Optimization

Explosion Problem
- Initial Sales Targets
- EXPLOSION
- Optimal Supply Request
- Suppliers

Implosion Problem
- Optimal Commitment to Sales
- SUPPLIERS
- IMPLOSION
- Supply Commitment
### Explosion

<table>
<thead>
<tr>
<th><strong>Given</strong></th>
<th>A set of $m$ products and $n$ components</th>
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<tbody>
<tr>
<td></td>
<td>Initial sales targets for each product over $T$ weeks</td>
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</table>

<table>
<thead>
<tr>
<th><strong>First Stage Problem</strong></th>
<th>Determine a set of optimal <strong>supply requests</strong> for each component over $T$ weeks</th>
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<tr>
<th><strong>End of First Stage</strong></th>
<th>Actual order configurations are realized</th>
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<tr>
<th><strong>Second Stage Problem</strong></th>
<th><strong>Allocate requested supply</strong> among products assuming that demand is equal to the initial sales targets, for all $T$ weeks.</th>
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<th><strong>Objective</strong></th>
<th>Maximize total expected profits</th>
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<td></td>
<td>Profit = Revenue – Cost (ordering, inventory, shortage)</td>
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Demand/Supply Optimization

Explosion Problem

- Initial Sales Targets
- EXPLOSION
- Optimal Supply Request

Suppliers

Implosion Problem

- Supply Commitment
- IMPLOSION
- Optimal Commitment to Sales

Optimal Supply Request
### Implosion

| Given | A set of \( m \) products and \( n \) components  
|       | Initial sales targets for each product over \( T \) weeks  
|       | Component supply commitments over \( T \) weeks |
| First Stage Problem | Determine a set of optimal **sales targets** for each product over \( T \) weeks |
| End of First Stage | Actual order configurations are realized |
| Second Stage Problem | **Allocate** committed supply among products assuming demand is equal to the optimal **sales targets**, for all \( T \) weeks. (May also consider supply flexibility) |
| Objective | Maximize total expected profits, less potential **penalties for deviating from the initial sales targets**. |
Solving 2 Stage SPs

- Backwards Induction
  - solve 2\textsuperscript{nd} stage, then 1\textsuperscript{st} stage, exactly
  - Not practical for realistically sized supply chain problems

- Simultaneously, using linear programming
- Iteratively, using decomposition methods
Constraint Matrix of Deterministic Equivalent for 2 Stage SP

Stage 1

Stage 2 (scenario 1)

Stage 2 (scenario 2)

Stage 2 (scenario 3)
Constraint Matrix of Deterministic Equivalent for 2 Stage SP
LP solvers

- Commercial
  - CPLEX
  - XPRESS
  - LINDO

- Open Source/Free
  - CLP (COIN-OR)
  - Soplex
  - GLPK
Decomposition Methods (in a nutshell)

- Multi-cut L-shaped method (Van Slyke & Wets 1969)
  - Solve only one scenario at a time
  - In each iteration
    - First solve first stage problem and obtain feasible solution $x$.
    - Then, solve $K$ scenarios of second stage
    - Add constraints to the first stage problem to improve approximation of second stage problem
    - Repeat until termination criteria is satisfied.

Can be parallelized.
Linderoth & Wright 2002
Decomposition Methods (in a nutshell)

- Stochastic Decomposition (Higle & Sen 1996)
  - Solve each scenario only once
  - In each iteration
    - Solve first stage problem and obtain feasible solution x
    - Solve 1 new scenario of second stage
    - Add constraints to the first stage problem to improve approximation of second stage problem
    - Repeat until termination criteria is satisfied
Research Objectives

- To demonstrate the value of using stochastic models to address the following Configure-To-Order supply chain problems on a realistic scale:
  - Determine optimal component supply request to support a set of initial sales targets (i.e., the ‘explosion’ problem).
  - Determine optimal product sales targets for a given component supply commitment (i.e., the ‘implosion’ problem).

- To incorporate flexibility in supply into our model.
Data Set

- Using data provided by IBM, we solved a 5 product 279 component problem. (a ‘realistic’ problem could include 100+ products and 3000+ components)
- The problem is solved for a 32 week planning horizon.
- Component usage quantities are sampled from empirical distributions created from historical order configuration data.
- The problem is modeled as a 2 stage stochastic optimization problem and is solved using CPLEX.
The Value of Stochastic Modeling

Comparison of Expected Cumulative Profit for Explosion Problem

Comparison of Expected Cumulative Profit for Implosion Problem
### Average Improvement Achieved Through the Use of Stochastic Models

<table>
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<tr>
<th>Definition</th>
<th>Explosion</th>
<th>Implosion</th>
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<tbody>
<tr>
<td><strong>Revenue</strong></td>
<td>0.04</td>
<td>-0.02</td>
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<tr>
<td>Revenue refers to the earnings from product sales.</td>
<td></td>
<td></td>
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<tr>
<td><strong>Backorder Costs</strong></td>
<td>0.78</td>
<td>0.71</td>
</tr>
<tr>
<td>Per unit backorder costs are incurred when the targeted product volume cannot be produced due to component shortages.</td>
<td></td>
<td></td>
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<tr>
<td><strong>Holding Costs</strong></td>
<td>0.21</td>
<td>-0.39</td>
</tr>
<tr>
<td>Per unit holding costs are incurred while excess components are held in inventory.</td>
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<tr>
<td><strong>Fill Rate</strong></td>
<td>0.35</td>
<td>0.24</td>
</tr>
<tr>
<td>The fill rate is the percentage of targeted sales that is fulfilled in a timely fashion.</td>
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</tr>
<tr>
<td><strong>Profit</strong></td>
<td>3.53</td>
<td>5.13</td>
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<tr>
<td>The difference between revenue earned and the sum of backorder and inventory costs.</td>
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Effect of Supply Flexibility

Cumulative Commitment to Sales Under Various Degrees of Supply Flexibility

- 0%
- 10%
- 20%
- Target

Weeks vs. Relative Quantity of Product A
Future Work

- Our results suggest that:
  - Dealing with configuration uncertainty is effective in improving expected profits.
  - Accounting for supplier flexibility can improve the commitment to sales.

- Our current work involves implementing decomposition methods for dealing with larger problem sizes.

- The hope is to be able to incorporate large-scale stochastic optimization tools within the existing integrated demand & supply planning framework at IBM.
Acknowledgements

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- Research collaborators:
  - Terry Harrison, Smeal College of Business
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  - Barun Gupta, IBM Integrated Supply Chain
Summary

- Supply chain management is central to the operations of many, if not all, goods and services providers.

- This presentation focused on a tactical planning problem (demand/supply planning) faced in configure to order supply chains.

- We address this problem using Stochastic Programming methods.

- Realistically sized problems require significant computational power and efficient LP solvers.