Application of Direct Simulations and 3-D Visualization to Evaluate the Influence of Mean Shear on the Dynamics of Turbulence*

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Turbulence Shear Flows: Structure

vortices within

Cumulonimbus with tornado
Vortices Within
Objectives

• Develop an analytical method to quantify and visualize concurrently local “structures”
  ➢ in vorticity, strain-rate and Reynolds stress

• Quantify and visualize the influence of mean shear on:
  ➢ the generation and evolution of structures in fluctuating vorticity
  ➢ the relationship between fluctuating strain-rate and vorticity structures

• Follow visually and quantitatively the birth, life and death of a hairpin vortex in shear flow
Methods

• DNS of homogeneous turbulent shear flow

• Algorithm for extracting “structures”

• Visualization (subjective), with local statistics

• Global statistics
DNS of Homogenous Turbulence

Isotropic Turbulence Simulation
256 × 128 × 128

Shear Turbulence Simulation
128 × 128 × 128

Initial Gaussian
N.-S. Eqn. ⇒ Fully Isotropic
Random Field Turbulence

$R^* \sim 22$

Shear Turbulence
$R^* \sim 65-75$

$S^* \sim$

$S^*$ vs $St$
Structure Extraction Concept

Basic Methodology

3-D Turbulent Data Set

Extraction of 3-D Intermittent Regions

Ordering by Peak Intensity

Grouping for Subsequent Analysis

Low-Magnitude Fluctuations Set

Intermittent Regions (High-Magnitude Set)
“Structure” Extraction
“Structure” Extraction

Interconnected Structures
Isosurface Structure vs. “Structures”
Isosurface Structure vs. “Structures”

\[ P(\omega^2) \]

\[ \omega^2 \]
Isosurface Structure vs. “Structures”

\[ P(\omega^2) \]
Isosurface Structure vs. “Structures”
Confirming the Hairpin
Enstrophy and Strain-rate Structures
Enstrophy and -uv Structures
Vorticity—Strain-rate Dynamics

\[
\frac{d\langle \omega^2 \rangle}{dt} = \langle P_{\omega^2} \rangle + \langle \overline{P}_{\omega^2} \rangle + \langle D_{\omega^2} \rangle
\]

\[
\frac{d\langle s^2 \rangle}{dt} = \langle P_{s^2} \rangle + \langle \overline{P}_{s^2} \rangle + \langle PR_{s^2} \rangle + \langle D_{s^2} \rangle
\]

\[
\langle \omega^2 \rangle = 2\langle s^2 \rangle
\]

\[
\langle P_{\omega^2} \rangle = 2\langle \omega_t s_{ij} \omega \rangle = -\frac{3}{2}\langle s_{ij} s_{jk} s_{ki} \rangle
\]

\[
\langle P_{s^2} \rangle = -\langle s_{ij} s_{jk} s_{ki} \rangle - \frac{1}{4}\langle \omega_t s_{ij} \omega \rangle = \frac{24}{13}\langle P_{\omega^2} \rangle
\]

In \( s_{ij} \) principle axes \( (\alpha, \beta, \gamma) \):

\[
P_{\omega^2} = 2(\alpha \omega^2_{\alpha} + \beta \omega^2_{\beta} - |\gamma| \omega^2_{\gamma})
\]

\[
\beta_n \equiv \beta/(s^2 / 6)^{1/2}
\]
Alignment by Mean Shear

P(γ) of highest intensity \( \omega^2 \) structures
Alignment by Mean Shear

Shear Tends to Produce Squashed Vortex Tubes
Global Two-Dimensionalization and Alignment of Strain-rate Fluctuations

**Isotropic:** \( \beta_n \sim 0.6 \)
\( \alpha: \beta: \gamma \approx 2.2 : 1 : -3.2 \)

**Shear:** \( \beta_n \sim 0.28 \)
\( \alpha: \beta: \gamma \approx 5.6 : 1 : -6.6 \)
“Passive” vs. “Active” Strain-rate Fluctuations

Passive strain-rate fluctuations

- locally two dimensional: \( s_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -c \\ 0 & c & 0 \end{bmatrix} \), \( \beta = 0 \)
- perfect alignment: \( \hat{e}_\beta \parallel \vec{\omega} \)
- no production: \( P_{\omega^2} = P_{s^2} = 0 \)

\( \Rightarrow \) passive \( s_{ij} \) are a kinematic consequence of concentrated vorticity
Active and Passive Enstrophy Structures

enstrophy production rate

\[ \langle P_{\omega^2} \rangle_{\text{structure}} = 2 \langle \omega_i \omega_j \rangle_{\text{structure}} \]
Active and Passive Enstrophy Structures

**Enstrophy Production Rate**

\[ \langle \mathbf{P} \omega^2 \rangle_{\text{structure}} = 2 \langle \omega_i s_{ij} \omega_j \rangle_{\text{structure}} \]

**Alignment with Vorticity**

\[ \langle \cos \theta_{\beta} \rangle_{\text{structure}} \]

**Magnitude of Second Eigenvalue**

\[ \langle \beta_n \rangle_{\text{structure}} \]

\[ \langle \mathbf{P} \omega^2 \rangle_{\text{structure}} \text{ structure number} \]
Creation of Horseshoe Vortices by Shear

**process:** identify same $\omega^2$ structure backwards/forwards in time from shear state
The Creation Process

$St = 0$

$St = 0.2$

$St = 0.4$

$St = 0.6$

$St = 0.8$

$St = 1.0$

$St = 1.2$

$St = 1.4$

$St = 1.6$

$St = 1.8$

$St = 2.0$

$St = 2.2$
The Initial Transitional Period

$St = 0$

$St = 0.2$

$St = 0.4$

$St = 0.6$

$St = 0.8$

$St = 1.0$

$St = 1.2$

$St = 1.4$

$St = 1.6$

$St = 1.8$

$St = 2.0$

$St = 2.2$
Shear Layer to Vortex Tube

$St = 0$

$St = 0.2$

$St = 0.6$

$\omega^2$ structure

$s^2$ isosurface
The Initial Creation Process

$St = 0$

$St = 0.2$

$St = 0.4$

$St = 0.6$

$St = 0.8$

$St = 1.0$

$St = 1.2$

$St = 1.4$

$St = 1.6$

$St = 1.8$

$St = 2.0$

$St = 2.2$
Later Evolution of Horseshoe Vortex

$St = 1.8$

$St = 2.0$

$St = 2.2$

$St = 2.4$

$St = 2.6$

$St = 2.8$

$St = 3.0$

$St = 3.2$

$St = 3.4$
Quantifying Horseshoe Vortex Evolution

\[ \text{corr}(\omega^2, s^2) \]

\[ \cos \theta_\beta \]
Summary

Structures
• concentrations of different fluctuating turbulence variables as coherent structures
  ⇒ concurrent quantification with visualization: evolution

Shear
• aligns and squashes vortex tubes
• two-dimensionalizes $s_{ij}$
• enhances passive $s_{ij}$
• enhancement is within the more intense enstrophy structures

Structures
• most enstrophy production is in the second $s_{ij}$ eigenvalue associated with the more intense enstrophy structures
  ⇒ both passive and active $s_{ij}$ are enhanced by shear