Collisions of Black Holes & Neutron Stars

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Black Holes

Fiction?  Reality?
Supermassive Black Holes

Quasars & Active Galaxies

\[ M \geq 10^6 M_\odot \]
Tidal Disruptions of Stars by Supermassive Black Holes
Solar Mass Black Holes

Credit: NASA/ESA
Neutron Stars & Pulsars

Crab Nebula
Gravitational Waves

- They interact very weakly with matter
- We will be able to see directly the source
- A new window in Astronomy will be opened
Gravitational Waves: ripples in the fabric of space-time
Interferometers

\[ L = 5 \text{ km} \]

\[ \Delta L \approx \frac{\text{proton radius}}{10,000} \]

\[ h = \frac{\Delta L}{L} \approx 10^{-23} \quad \text{Wave Amplitude} \]
Earth-based Interferometers
The Ultimate Test of General Relativity: Binary Black Hole and/or Neutron Star Collisions

**Numerical Relativity:** Numerical solutions to Einstein’s equations

\[ G_{\mu\nu} = 8\pi T_{\mu\nu} \]
First Grand Challenge

The Grand Challenge Equations

\[ B_i A_i = E_i A_i + \rho_i \sum_j B_j A_j F_{ji} \]

\[ \nabla \times \vec{B} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \vec{F} = m \vec{a} + \frac{dm}{dt} \vec{v} \]

\[ dU = \left( \frac{\partial U}{\partial S} \right)_V dS + \left( \frac{\partial U}{\partial V} \right)_S dV \]

\[ \nabla \cdot \vec{D} = \rho \]

\[ Z = \sum_j g_j e^{-E_j/kT} \]

\[ F_j = \sum_{k=0}^{N-1} F_{kj} e^{2\pi i j k / N} \]

\[ \nabla^2 u = \frac{\partial u}{\partial t} \]

\[ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \]

\[ \nabla \cdot \vec{B} = 0 \]

\[ p_{n+1} = r p_n (1 - p_n) \]

\[ -\frac{h^2}{8\pi^2 m} \nabla^2 \Psi(r,t) + \nabla \Psi(r,t) = -\frac{h}{2\pi i} \frac{\partial \Psi(r,t)}{\partial t} \]

\[ -\nabla^2 u + \lambda u = f \]

\[ \frac{\partial u}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\frac{1}{\rho} \nabla p + \gamma \nabla^2 \vec{u} + \frac{1}{\rho} \vec{F} \]

\[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f \]

- Newton's Equations
- Schrödinger Equation (Time Dependent)
- Navier-Stokes Equation
- Poisson Equation
- Heat Equation
- Helmholtz Equation
- Discrete Fourier Transform
- Maxwell's Equations
- Partition Function
- Population Dynamics
- Combined 1st and 2nd Laws of Thermodynamics
- Radiosity
- Rational B-Spline
Einstein Equations & Geometrodynamics

\[ G_{\mu\nu} = 8\pi T_{\mu\nu} \]

Space-Time Geometry = Matter-Energy

John A. Wheeler

Geometrodynamics: Time history of the Geometry of Space
Formulations of the Einstein Equations

\[ G_{\mu\nu} = 8\pi T_{\mu\nu} \quad \text{and} \quad G_{\mu\nu} = F[\partial^2 g_{\alpha\beta}, (\partial g_{\alpha\beta})^2, \ldots] \]

**Analogy:**

\[ -\partial_{tt}\Phi + \partial_{xx}\Phi + \frac{1}{\Phi} (\partial_t\Phi)^2 = 8\pi \rho \]

**One possibility:**

\[ \partial_t\Phi = K \]

\[ \partial_tK = \partial_{xx}\Phi - \frac{K^2}{\Phi} - 8\pi \rho \]

**Another possibility:**

\[ \partial_t\Phi = -K \]

\[ \partial_tK + \partial_x M = -\frac{K^2}{\Phi} - 8\pi \rho \]

\[ \partial_tM + \partial_x K = 0 \]
Instabilities!

Constraint violating modes?
What?

\[
\begin{align*}
\partial_t \vec{A} & = -\vec{E} - \nabla \Phi \\
\partial_t \vec{E} & = -\nabla^2 \vec{A} + \nabla (\nabla \cdot \vec{A})
\end{align*}
\]

Evolution equations

\[\nabla \cdot \vec{E} \equiv C = 0\]

Constraint

At analytic the level:

\[C(t = 0) = 0 \quad \Rightarrow \quad C(t) = 0\]

Not the case at the discrete level!
Second Grand Challenge

GIGO = Garbage In, Garbage Out
Initial Data Problem

Goal: Construct astrophysically relevant data satisfying the Einstein Constraints
E&M Analogy

\[
\begin{align*}
\partial_t \vec{A} &= -\vec{E} - \nabla \Phi & \text{Evolution equations} \\
\partial_t \vec{E} &= -\nabla^2 \vec{A} + \nabla (\nabla \cdot \vec{A}) \\
\nabla \cdot \vec{E} &= 0 & \text{Constraint}
\end{align*}
\]

Initial Data: \((\vec{E}, \vec{A})\) 6 quantities but only 1 equation!

Clearly \(\vec{A}\) is freely specifiable.

But, which of the 3 components of \(\vec{E}\) is fixed by \(\nabla \cdot \vec{E} = 0\)

\[
\vec{E} = \vec{T} + \nabla \phi \\
\nabla \cdot \vec{E} = 0 \quad \iff \quad \nabla^2 \phi = 0
\]

Transverse + Longitudinal
Third Grand Challenge

The Black Hole Singularity: When Nature Divided by Zero
Another Analogy

Subsonic

Supersonic
Black Hole Excision

Excision
Unruh (1984)
Not that easy!

\[
\frac{\partial \phi}{\partial x}_{i,j} = \frac{\phi(x_{i+1}, y_j) - \phi(x_{i-1}, y_j)}{\Delta x}
\]

\[
\frac{\partial \phi}{\partial y}_{i,j} = \frac{\phi(x_i, y_{j+1}) - \phi(x_i, y_{j-1})}{\Delta y}
\]
Finite-Differences at the Excision Boundary
High Performance Computing

Scales:

• Characteristic scale M (BH mass)
• Minimum resolution ~ M/20
• Radiation wavelength ~ 15 M
• Computational domain ~ 500 M
• Evolution time ~ 500 M
• Floating point operations per grid-point ~ $10^5$
PSU Numerical Relativity Gang

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Single Black Holes
Head-on BH Collisions
BH Orbits and Merger

Lapse $\alpha$ at $T=0.0M$
Black Hole - Neutron Stars
The Maya Code

- PDEs
- Kranc
- Cactus
- Maya
- Carpet

Einstein Equations
Mathematica scripts to generate the source
Parallelization, IO, Time Updates, Grid-functions, Parameters
www.cactuscode.org
Tools
Adaptive mesh refinements infrastructure
Horizon trackers, wave extraction, etc.
Conclusions & Future

- Black Hole - Neutron Star Binaries are one of the most important sources of gravitational radiation.

- Gravitational waves will be detected within 10 years.

- A new window will be opened: Gravitational Wave Astronomy.

- Numerical simulations will play a crucial role in the detection and characterization of source of gravitational waves.